JPL

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To: Distribution

From: Robert A. Mase

Subject: Update to Mars Coordinate Frame Definitions

This memo is an update to the Mars Observer Planetary Constants and Models document, published November 1990. Specifically, to Section 2.8 which deals with Mars centered coordinate frame definitions and relationships.

Introduction

This memo was undertaken to update some of the information that has grown stale in the Mars Observer Planetary Constants and Models document (Ref 1). This work was undertaken specifically for MSP'01 mission planning, but the results apply generically to all Mars missions. The MO document has become the de-facto reference for all of the recent JPL Mars missions, so it is appropriate to update it such that all interested parties may be assured they are using coordinated and correct information. The focus here will be on Mars centered coordinate systems, and only a subset of those systems will be described. This memo is not intended to be a generic reference on the subject, merely an update to certain coordinate frame definitions that are of interest to Mars missions.

The first part of this memo will briefly define the terminology that is customarily utilized in the description of these systems. The definitions presented are either based on or taken verbatim from the reference material. Then, several specific Mars relative coordinate systems will be carefully defined. The process used to define these systems remains unchanged from the MO document, however, certain quantities associated with those systems will be updated. Finally, the relationships between the systems of interest will be calculated.

Basic Definitions

Any treatment of astrodynamic quantities will involve the use of coordinate systems. A coordinate system is simply a means of relating points in three dimensional space, and their motion as ordered triples (ex. X,Y,Z). The system is fully specified by three fundamental characteristics: frame, center and type.

Frame

A ***coordinate frame*** represents an associated set of Cartesian axes: X,Y,Z which are themselves specified by four additional items: reference body, reference plane, reference direction, and reference time.

The ***reference body*** can be any body in the solar system. For the applications of this memo, the reference body will be either the Earth or Mars. Note that the specification of a reference body name in the coordinate frame definition says nothing about where the frame is actually centered. For example, we will later define an Earth equator frame centered at Mars.

The ***reference plane*** along with the reference body define the X-Y plane of the frame. Two pertinent examples of reference planes are the Earth equator plane and the Earth orbit plane. Because a plane is defined by its unit normal, the Z-axis defines the X-Y reference plane. For an equator plane, the Z-axis will generally be along the axis of rotation of the planet. For an orbit plane, the Z-axis will be in the direction of the orbital angular momentum vector.

The ***reference direction*** is an arbitrary, although usually physically meaningful, direction in the X-Y plane that defines the direction of the X-axis. Typically, such a direction is obtained from the line of nodes that result from the intersection of the reference plane with some other known plane. For example, the X-axis of the Earth equator plane may be the vernal equinox, which is the ascending node of the Earth orbit plane on the Earth equator plane. This system is called the Earth Equator and Equinox plane for obvious reasons. In another case, the X-axis of the Mars equator plane may be the ascending node of the Mars equator plane on the Earth equator plane, illustrating the fact that the two planes do not necessarily have to have the same reference body name. This system is called the Mars Equator and IAU-vector plane since the X-axis definition is that specified by the International Astronomical Union (Ref 2). A frame with a body-fixed X-axis will rotate as the body rotates. Such a system would typically utilize the intersection of the body's prime meridian with the equator plane to define the direction of the X-axis. Such a system is defined for Mars and is called the Mars Equator and Prime Meridian plane.

The ***reference time*** is a specification of when the frame being described has, had or will have a physical existence. It is necessary to specify a reference time, because the reference planes that describe the coordinate frames are likely to be in some state of motion due to the fact that the bodies they are associated with are continually being subjected to the perturbing forces of the physical universe.

For the Earth, the plane of the ecliptic and the plane of the equator are used as planes of reference, and their intersection, the equinox, is in a state of motion due to their motion. The motion of the ecliptic plane is due to the the gravitational action of the planets on the Earth's orbit and makes a contribution to precession known as ***planetary precession.*** The motion of the equator is due to the torque of the Sun, Moon, and planets on the oblate Earth. This motion can be divided up into two parts, ***lunisolar precession***, which is the smooth, long-period motion of the ***mean pole*** of the equator about the pole of the ecliptic, with a period of about 26,000 years, and ***nutation***, which is the short-period motion of the ***true pole*** around the mean pole with a variety of periods up to 18.6 years. The combination of lunisolar and planetary precession is called ***general precession.***

Precession and nutation taken together represent the ***true*** motion of a reference plane, as well as its Z-axis, which manifests as a cyclic behavior about a ***mean*** motion that would be obtained if precession were considered alone. Thus the Earth *True* Equator plane reflects the effects of precession and nutation, whereas the Earth *Mean* Equator plane reflects the effect of precession alone.

Both of these influences are related to the concept of inertial and non-inertial reference frames. An ***inertial*** frame is one in which the Cartesian axes are in a state of rest or uniform (unaccelerated) motion. Since rotation always involves acceleration, rotating systems cannot be considered inertial. A ***non-inertial*** system is one which is undergoing some sort of non-uniform (accelerated) motion and is most commonly thought of as one in which the axes are rotating. A body-fixed frame is a specialized type of non-inertial frame, though it is certainly not the only such frame.

The most common type of motion for axes in a non-inertial frame is rotation, although precession and nutation are also examples. The reference time defining the orientation of the axes is taken to be the same time at which a state is to be related to the frame. This reference time is considered to be ***"of-date"***. One example of a non-inertial frame is the Earth Mean Equator and Equinox of Date, which reflects only the precession of the Earth's pole. However, it is noteworthy that this system is not too different from an inertial system since precession is a long period motion. Another non-inertial frame is the Earth True Equator and Equinox of Date, which reflects both the precession and nutation of the Earth pole, the latter of which is of short period and more rapid. Finally there are the ***body-fixed*** frames such as the Earth True Equator and Prime Meridian of Date, which is non-inertial because, in addition to precession and nutation, it reflects the daily rotation of the X-axis which is tied to the Prime Meridian.

In order to specify an inertial frame, it is typical, though not essential, to begin with a mean equator or mean orbit plane (i.e. one which considers only precession) and freeze it at a particular instant of time. This moment, or reference time, is called the ***epoch*** of the reference plane and represents a snapshot of the position of the plane at that instant in time. The standard reference epoch is 01-Jan-2000 12:00:00 ET, commonly called J2000. This is the beginning of the Julian year 2000, and corresponds to a Julian date of 2451545.0. The fundamental inertial frame definition uses the Earth as the reference body, its mean equator as the reference plane, the vernal equinox of its mean orbit as the reference direction, and J2000 as the reference epoch. Hence, this frame is called the Earth Mean Equator and Equinox of Epoch J2000 or simply EME2000.

Center

The ***center*** of a coordinate frame is the origin of the system. It may be at the center of any of the nine planets, their natural satellites, comets, the Sun, the solar system or any planet system barycenter, at a topographic location on any of these bodies, or at a spacecraft. The reference body name used in the frame definition is not necessarily related to the body center, although frequently the reference body is chosen as the frame center. One such system would be the Mars-centered Mars Mean Equator and Equinox of Date. Another example that is widely used for interplanetary navigation is a Sun-centered Earth Mean Equator and Equinox of Epoch J2000. In this case, the inertial frame is related to the Earth, while the system is centered at the Sun.

Type

Given a center and a coordinate frame, there are then several ***types*** of coordinates commonly used to represent state vectors. They are all equivalent, and each requires the specification of six parameters to specify both the position and velocity of a point in the frame. Because this memo is of a limited scope, only cartesian and spherical coordinates will be discussed here.

Cartesian coordinates consist of the X, Y, and Z triple to specify position and the time derivatives of these to specify velocity. These are most useful during the process of numerical integration of a trajectory, although they may give little insight into the overall characteristics of the trajectory.

Spherical coordinates utilize a distance (radius or altitude), and two angles (declination and right ascension, or latitude and east longitude) to specify position, and speed, flight path azimuth and flight path angle to specify velocity. Generally declination and right ascension are used with an equator plane when the X-axis is not rotating diurnally, such as an equinox X-axis. One example is the Earth-centered Earth Mean Equator and Equinox of Epoch J2000. In this case right ascension is measured along the mean equator, positive East (in the right-handed sense) from the equinox. Declination is measured from the mean equator plane positive to the North.

Latitude and longitude are commonly used for an equator plane with an X-axis that rotates diurnally, such as a Prime Meridian based system. These coordinates are useful in relating the motion of a spacecraft to a central body. An example is the Earth-centered Earth Mean Equator and Prime Meridian of Date. In this case, East longitude is measured along the mean equator positive East from the Prime Meridian. Latitude is measured from the mean equator positive North.

In addition, latitude and longitude are sometimes used when the reference plane is in a body's orbit, even though the X-axis of such a system is not generally rotating diurnally. One example is Sun-centered Earth Mean Orbit and Equinox of Epoch J2000 spherical coordinates, sometimes referred to as Celestial coordinates. In this case Celestial Longitude is measured in the plane of the ecliptic positive East (in the right-handed sense) from the equinox. Latitude is measured from the ecliptic plane positive to the North.

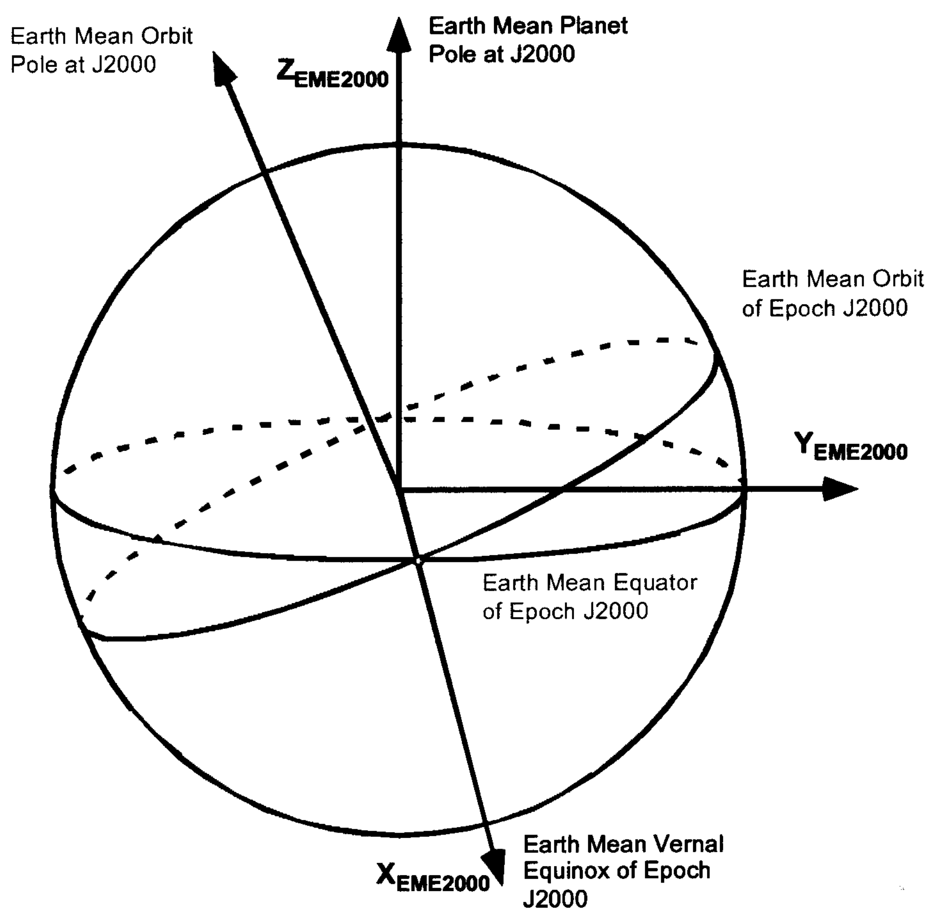
The EME2000 Inertial Reference System

The Earth Mean Equator and Equinox of Epoch J2000 inertial reference system is a right-handed Cartesian set of three orthogonal axes chosen as follows:

$+Z_{EME2000}$	is normal to the Earth mean equator at epoch J2000
$+X_{EME2000}$	is parallel to the vernal equinox of the Earth mean orbit at J2000
$+Y_{EME2000}$	completes the right-handed system.

The epoch J2000 is the Julian Ephemeris Date (JED) 2451545.0. The coordinate system is shown in the following figure. The coordinate frame need not be centered at the Earth, but for illustrative purposes it is easier to visualize. This reference frame can also be referred to as EME2000.

Figure 1: EME2000 Inertial Reference System



Mars-Centered Non-Inertial Coordinate Systems

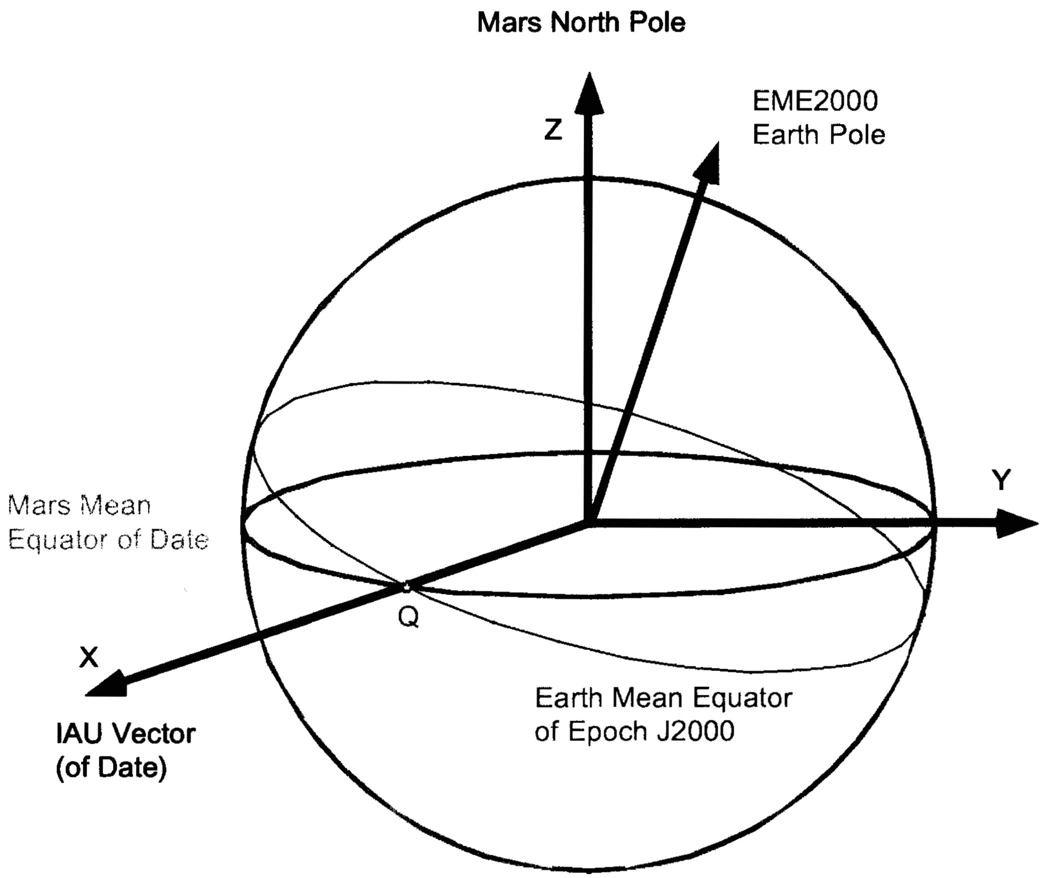
When a spacecraft is in the vicinity of Mars, it is convenient to utilize Mars-centered coordinate systems. These are systems that are centered at the center of the planet itself, as opposed to the system barycenter or on the planet surface. The systems described here are utilized regularly by the flight operations and mission planning teams for JPL Mars missions. The various institutions that work with JPL on these missions prefer to use at least one of these systems, and a clear definition and method of rotating between them is necessary. Because most of the working coordinate systems are, strictly speaking, non-inertial, these will be described first, and their inertial counterparts will be described later for completeness.

Mars-centered Mars Mean Equator and IAU-vector of Date

This coordinate system is the standard system for specification of most Mars relative states. In this system, Mars is the reference body, the Mars mean equator of date is the reference plane, the IAU-vector is the reference direction, the reference time is *of date*, and the system is centered at Mars. The IAU-vector is defined to be along the intersection of the Mars Mean Equator of Date plane and the Earth Mean Equator of Epoch J2000 plane, and is positive in the direction of the node Q. The node Q is defined as the ascending node of the Mars mean equator of date on the Earth mean equator of epoch J2000 (see Figure 2).

In the past, for coordinate systems in which the reference plane was the Mars mean equator, the reference direction was usually defined to be the Mars vernal equinox direction. However, the IAU does not compute planetary ephemerides, and only the Earth's mean orbit plane is known with great precision. The IAU therefore chose a reference direction that was not dependent upon the planet orbit, but was still in the mean equator plane. This direction is referred to as the IAU-vector, and although it has no physical meaning it is widely accepted because it is defined soley by IAU specified quantities.

Figure 2:
Mars-Centered Mars Mean Equator and IAU-Vector of Date
(Shown at J2000 epoch)

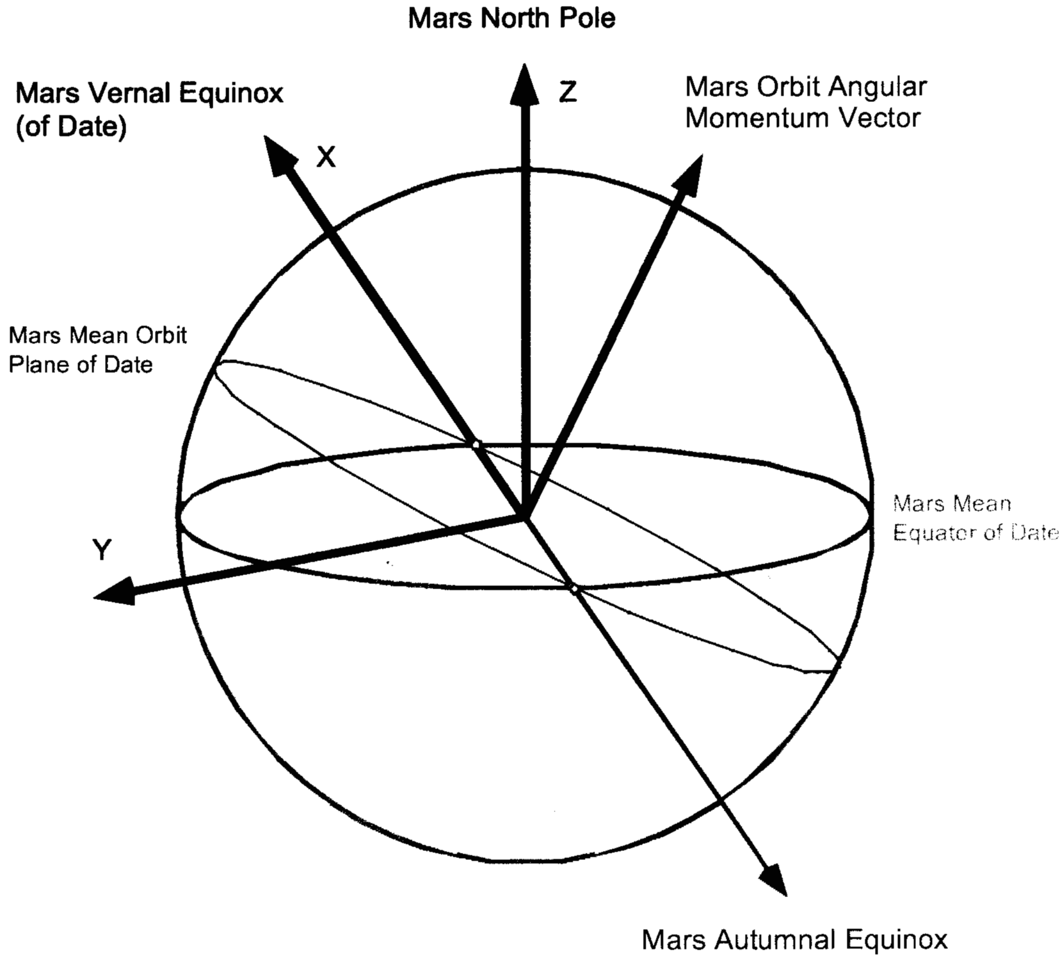


Mars-centered Mars Mean Equator and Equinox of Date

This system utilizes the same reference plane as the Mars-centered Mars Mean Equator and IAU-vector of Date coordinate system, but defines the reference direction to be along the Mars vernal equinox. As described above, the IAU recommends that equinox relative coordinate systems not be used for planets other than the Earth. However, this system is retained by some of the mission elements, so the definition and relationship are provided here.

Mars is the reference body, the reference plane is the Mars mean equator of date, the vernal equinox of the Mars mean orbit is the reference direction, the reference time is *of date*, and the system is centered at Mars. The equinox is defined as the intersection of the Mars mean orbit plane of date and the the Mars mean equator of date. The vernal equinox is in the direction of the ascending node of the mean orbit of date on the mean equator of date. This system is illustrated in Figure 3.

Figure 3:
Mars-Centered Mars Mean Equator and Equinox of Date
(Shown at J2000 epoch)



Mars-Centered Mars Mean Equator and Prime Meridian of Date
(based on the IAU-vector X-axis)

This frame is fixed in and rotates with Mars. The reference plane is the Mars mean equator, and the reference direction is the intersection of the Mars prime meridian with the reference plane. The prime meridian of Mars has been chosen to pass through a crater named Airy-0 located in the Southern hemisphere. The IAU specifies the location of the prime meridian by the angle ***W*** that is measured along the planet equator positive east (in a right-handed sense) from the node Q to the point where the prime meridian crosses the planet equator (See Figure 4). Again, the node Q is the ascending node of the planet equator of date on the Earth mean equator of epoch J2000.

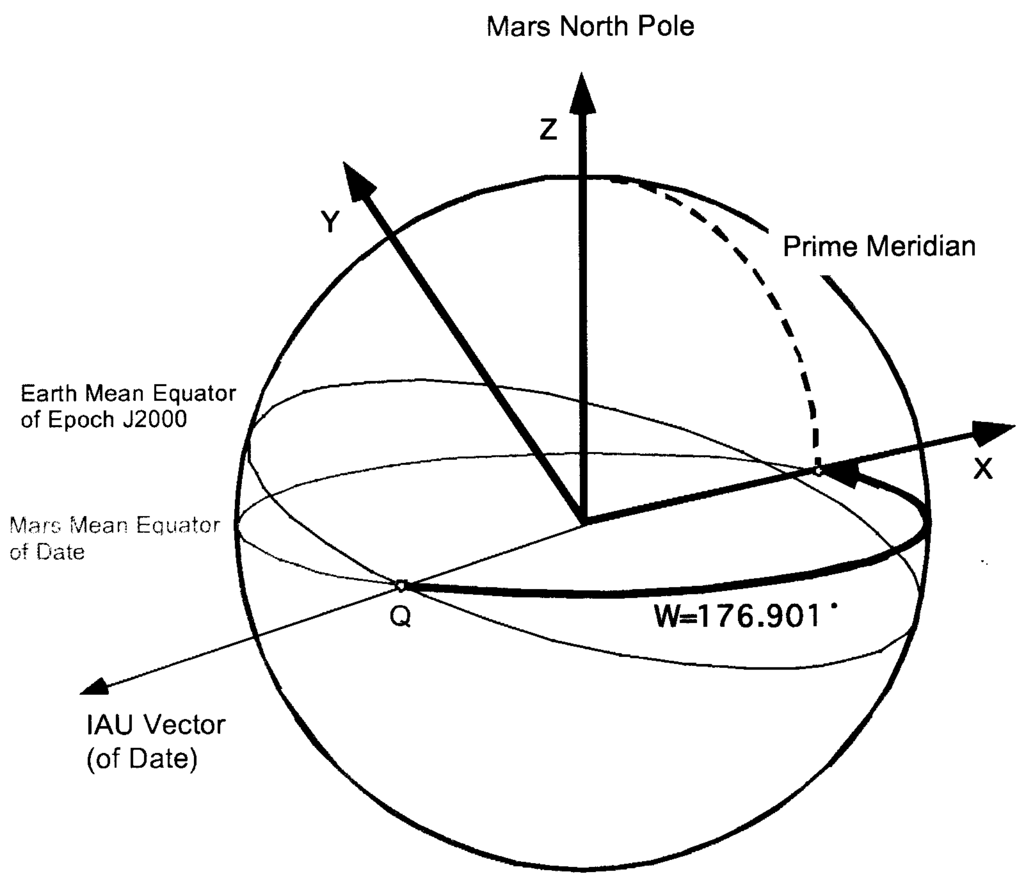
The angle W is specified by the following equation (Ref 2):

$$\mathbf{W = 176.901 + 350.8919830}\, \boldsymbol{d} \quad \mathbf{(deg)}$$

where the zeroth-order term denotes the right ascension of the prime meridian (with respect to the IAU-vector) at the J2000 epoch, and the first-order term provides for the rotation rate with respect to the IAU-vector, and *d* is measured in days (24 hour Earth days) past the J2000 epoch.

Figure 4 illustrates the frame definition and location of the prime meridian at the J2000 epoch.

Figure 4:
Mars-Centered Mars Mean Equator and Prime Meridian of Date
(shown at J2000 epoch)



Mars-Centered Inertial Coordinate Systems

As stated earlier, the axes of an inertial coordinate system must be fixed at an instant in time referred to as the *epoch* of the system. The epoch should be close enough in time to the dates of interest such that the effects of precession and nutation are negligible. The epoch J2000 is a convenient and logical choice for missions occurring at the start of the millennium, but the choice is arbitrary.

Mars-centered Earth Mean Equator and Equinox of Epoch

This inertial coordinate system is formed by attaching the Earth Mean Equator and Equinox of Epoch reference system to the center of Mars. The coordinate frame is well-defined: the Earth is the reference body, the Earth mean equator is the reference plane, the vernal equinox of the Earth mean orbit is the reference direction, the reference epoch is arbitrary but is usually chosen to be the J2000 epoch, and the system is centered at Mars. Since this is a fundamental reference system, it is useful in defining other coordinate systems at Mars.

Mars-centered Mars Mean Equator and IAU-vector of Epoch
Mars-centered Mars Mean Equator and Equinox of Epoch
Mars-centered Mars Mean Equator and Prime Meridian of Epoch

These are similar to the non-inertial coordinate systems described in the previous section, and can even have the same epoch as the date of the non-inertial systems. However, inertial planes would be frozen at that epoch, whereas the non-inertial planes would not be. At that instant, they would have the same orientation in space, although the coordinate axes of the non-inertial systems themselves would be in a state of rotational motion. Of course this motion, which is due to the precession of the Mars pole, is slight and could probably be ignored in most practical applications. Nevertheless, to be strictly accurate, it is necessary to distinguish between them.

Relating the Inertial, Non-Inertial and Body-Fixed Systems

Because each of the coordinate systems described above are used by the various elements within the mission design teams and flight projects, it is necessary to define the relationship between them. This will ease the interface process of rotating a state vector between coordinate frames.

The EME2000 reference frame is well defined, and all other frames can be described in relation to it. Again, this memo is focused on a subset of the inertial Mars-centered frames, namely Mars Mean Equator and Equinox of Epoch, Mars Mean Equator and IAU-Vector of Epoch, and Mars Mean Equator and Prime Meridian of Epoch. Since the reference plane for each of these three systems is the Mars mean equator, this implies that they all share the same Z-axis (at the same epoch). Therefore, the only difference between them is the definition of the X-axis, and each frame can be related to each other by a single rotation about the common Z-axis (Mars pole).

So an arbitrary point in space expressed in spherical coordinates will have the same radius and declination (or latitude) in all three systems, while the right ascension (or longitude) will be different. It is then sufficient to specify the angular difference, along the Mars mean equator, of the X-axis for each system to describe the relationship between them at a given epoch. Since the systems are inertial, the velocity component will not be corrupted by a rotating coordinate frame.

Definition of Mars Pole

The first step is to define the direction of the Mars pole. The mean pole (of date) direction for Mars is specified by the IAU (Ref 2) relative to the EME2000 reference frame. It is given in terms of right ascension and declination of the north pole as a function of time past the J2000 epoch by the following equations:

$$\alpha = 317.681 - 0.108\,T \quad \text{(deg)}$$
$$\delta = 52.886 - 0.061\,T \quad \text{(deg)}$$

Where T is time in Julian centuries of 36525 days past the reference epoch of J2000, JED 2451545.0.

So at the epoch of J2000, the pole direction can be converted into Cartesian coordinates to produce the Mars pole vector (of date) at the epoch J2000:

```
mars pole  =  (0.446161, -0.406246,  0.797437)
```

(Note: all cartesian vectors in this memo are normalized to produce unit vectors)

Calculation of the IAU-Vector

Once the pole of Mars is defined, it is then straightforward to calculate the direction of the IAU-vector (of date). As was shown in Figure 2, the IAU-vector lies along the intersection of the Mars mean equator of-date plane and the Earth mean equator of epoch J2000 plane. This direction can be computed as the cross product between the EME2000 pole vector and the Mars planet pole vector. The EME2000 pole vector is simply:

```
j2000 pole  =  (0, 0, 1)
```

Now the IAU vector (of date) is the cross product::

```
iau vector  =  j2000 pole X mars pole

iau vector  =  (0.673258, 0.739408,  0.000000)
```

By definition the IAU-vector is normal to the J2000 pole vector and the Mars pole vector, and the calculation bears this out. Recall that the IAU-vector is not an inertially fixed direction, as it is a function of the rotating Mars pole. But at a given epoch, the IAU-vector is the defining X-axis direction and the Mars prime meridian and equinox can be defined with respect to it.

Relating the Prime Meridian to the IAU-Vector

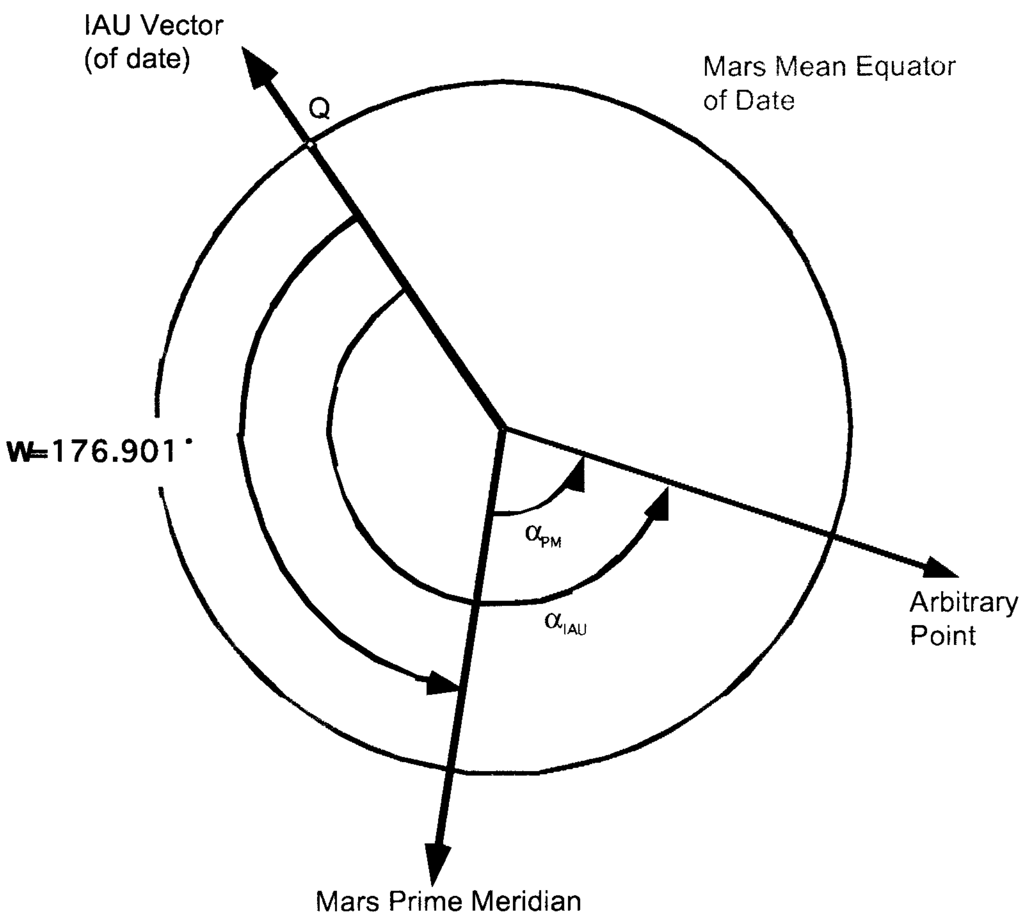
Recall that the IAU defines the prime meridian mathematically by specifying the angle W that is measured along the Martian equator from the node Q (IAU-vector) to the point where the prime meridian crosses the Martian equator.

$$\mathbf{W = 176.901 + 350.8919830}\, \boldsymbol{d} \quad \textbf{(deg)}$$

where the zeroth-order term denotes the right ascension of the prime meridian (with respect to the IAU-vector) at the J2000 epoch, and the first-order term provides for the rotation rate with respect to the IAU-vector (of date), and *d* is measured in days past the J2000 epoch.

So an arbitrary point in space specified in terms of the Mars Mean Equator and Prime Meridian of Date reference frame, can be converted to the Mars Mean Equator and IAU-vector of Date system by rotating the right ascension through the angle W. Figure 5 illustrates this at the epoch of J2000.

Figure 5:
Relationship Between Prime Meridian and IAU-Vector
(shown at Epoch J2000)



Right Ascension of Arbitrary point:

$$\alpha_{IAU} = \alpha_{PM} + W$$

Where:

W = 176.901 + 350.8919830*d* (deg)

d = days past J2000 epoch

Relating the Equinox of Mars to the IAU-Vector

To define the Mars vernal equinox direction, the mean orbit of Mars must be specified. This can be accomplished by specifying the mean orbit pole, or mean orbit angular momentum vector. The latest planet ephemeris, DE405, will provide the position and velocity of Mars with respect to the Sun at the J2000 epoch. The Mars orbit angular momentum vector can then be computed as the cross product: H = R X V, where H defines the orbit pole. Strictly speaking, this method will produce the osculating orbit of Mars, not the mean orbit. However, this assumption will have little effect on the coordinate transformations (see appendix).

Position and velocity of Mars in Sun-centered EME2000 cartesian coordinates, based on DE405:

```
mars position  =  (0.999647, 0.001007, -0.026567)
mars velocity  =  (0.044163, 0.908515,  0.415513)

orbit pole  =  position X velocity

orbit pole  =  (0.024569, -0.416780, 0.908675)
```

Now that the orbit pole of Mars is defined, the Vernal Equinox is defined as cross product between the Mars planet pole (of-date, previously defined at the epoch of J2000) and the orbit pole:

```
vernal equinox  =  mars pole X orbit pole

vernal equinox  =  (-0.086432, -0.906432 -0.413414)
```

Now an angle Ψ can be defined as the angle between the Vernal Equinox and the IAU-vector (at epoch J2000), and can be calculated via the dot product of the vectors.

$$\cos\Psi = \text{iau vector} \bullet \text{vernal equinox} = -0.728413$$

$$\Psi = 136.7535 \text{ deg}$$

Figures 6 and 7 illustrate the relationship between the two frames at the epoch J2000. The angle Ψ is measured positive eastward along the Mars mean equator from the vernal equinox to the IAU-vector.

The MO Constants document (Ref 1) defines the angle Δ as the angle between the IAU vector and the Autumnal Equinox. This quantity is more commonly found in the literature, and has a mathematical significance in astrodynamic geometry. However, for the purposes of this memo, it was easier to define and work with the supplement of this angle, which is arbitrarily defined to be Ψ.

Figure 6:
Relationship Between IAU-Vector and Mars Equinox
(shown at J2000 epoch)

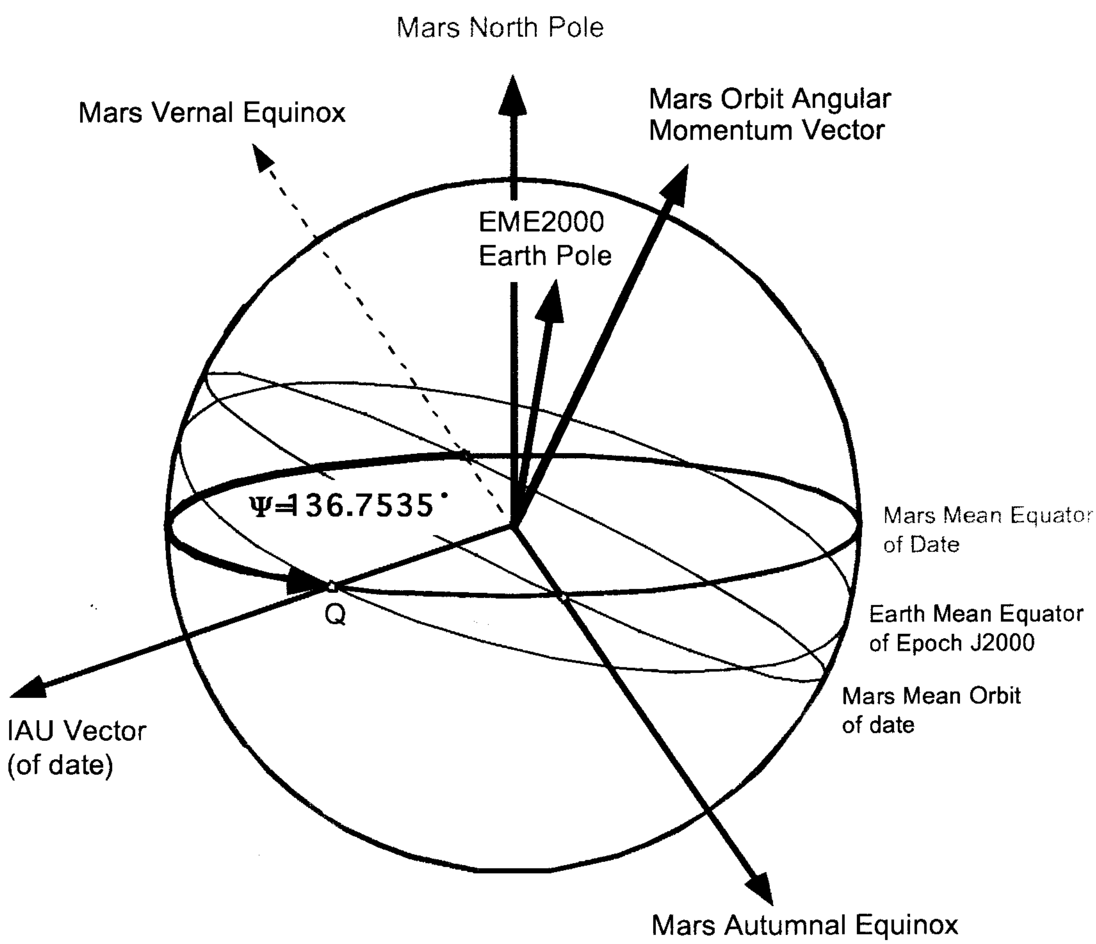
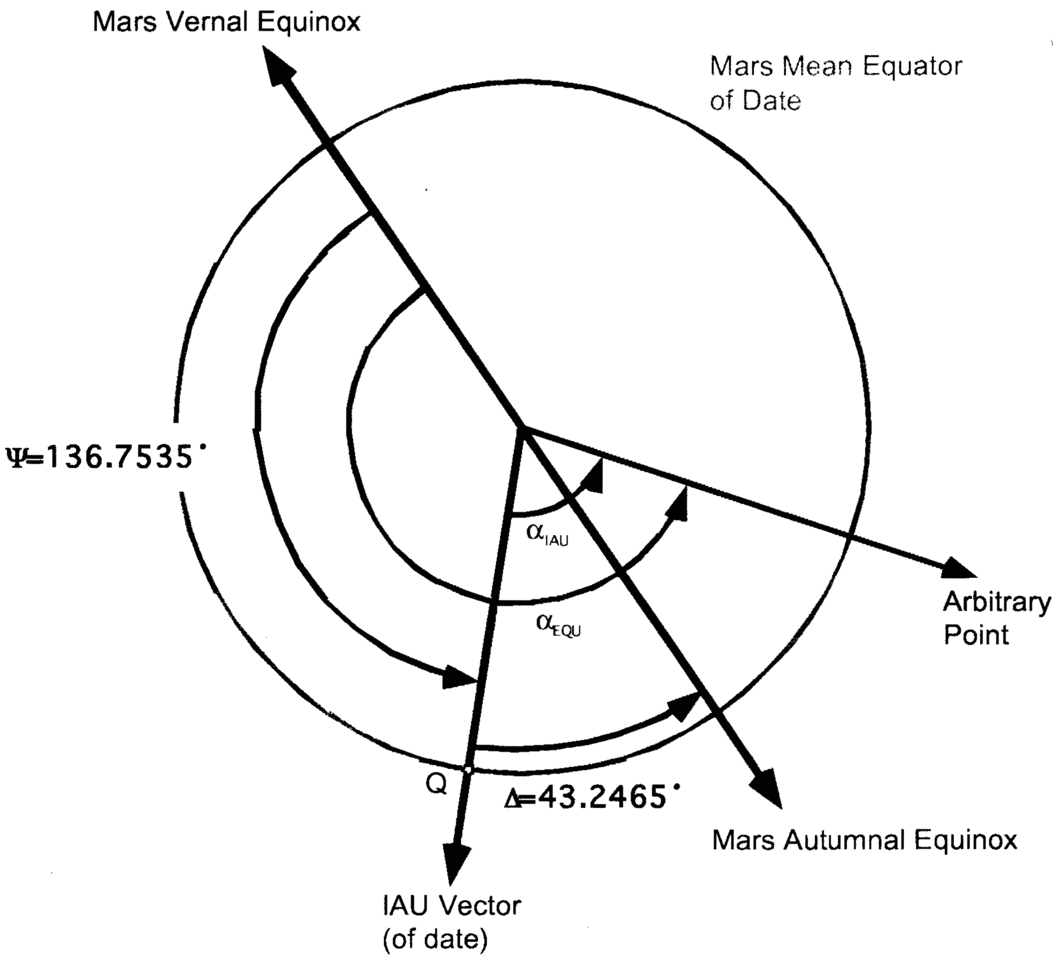


Figure 7:
Relationship Between IAU-Vector and Mars Equinox
(shown at J2000 epoch)



$$\alpha_{EQU} = \alpha_{IAU} + \Psi$$

The angle Δ is simply the supplement of Ψ and at the epoch J2000 has a value of:

$$\Delta = 180 - \Psi = 43.2465 \text{ deg}$$

Note, that this value of Δ is different from and supersedes the value presented in the original MO Planetary Constants document. This parameter is presented in Figure 7.

Relating the Equinox of Mars to the Prime Meridian

Now that the Prime Meridian and Equinox have both been defined with respect to the IAU-vector, they can now be related to each other. The prime meridian is measured from the IAU-vector by the angle W, and the angle Ψ measures the angle from the Equinox to the IAU-vector. We can now define the angle V, as the angle from the Vernal Equinox to the Prime Meridian as the sum:

$$V = W + \Psi$$

The IAU has defined W to be a function of time, but we have yet to determine a rate of change term for Ψ. So an additional relationship that must be defined is the rate at which the Mars equinox rotates with respect to the IAU-vector. Both are a function of the rotating Mars pole, and the equinox is also a function of the orbit pole which is also rotating. Therefore, the two axes rotate with respect to one another at a slow rate.

The Mars pole direction has been previously defined as a function of time. But again, as there is no accepted reference for the Mars mean orbit, no closed form solution exists to describe the motion of the Mars orbit pole. So instead of attempting to calculate a constant rate of change term (as is done in Reference 1), it is appropriate to calculate the actual value of Ψ at various epochs. Therefore, based on the DE405 osculating orbit of Mars at each epoch, Table 1, on the following page presents the value of Ψ calculated at various epochs over the coming decade that may be of interest to Mars missions.

So now that we have tabulated Ψ as a function of time, we can complete the calculation of the parameter V. Near the epoch of J2000, we can use the value of:

$$\Psi = 136.7535 \text{ deg}$$

and recall the equation for W:

$$W = 176.901 + 350.8919830\, d \quad \text{(deg)}$$

So:

$$V = W + \Psi$$

$$V = (176.901 + 350.8919830\, d) \text{ deg} + 136.7535 \text{ deg}$$

$$V = 313.6545 + 350.8919830\, d \text{ deg}$$

This equation is valid near the epoch of J2000. For other epochs, it is appropriate to use Table 1, below, to choose (or interpolate) a value of Ψ for the epoch of interest.

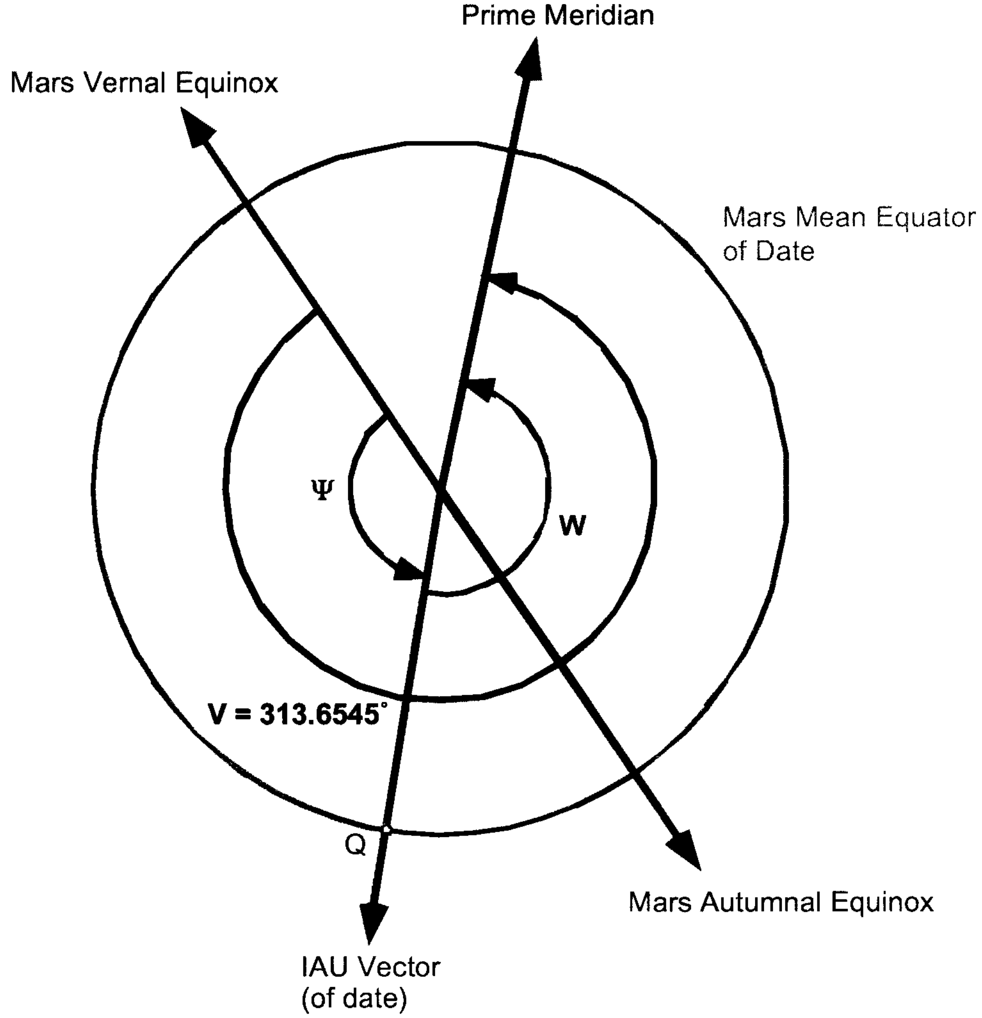
Figure 8 on the following page illustrates this relationship at the epoch of J2000.

Table 1: Calculation of Ψ as a Function of Time

Epoch	JD	Ψ (deg)	Relevance
01-JAN-1999 12:00:00 ET	2451180.0	136.7524°	
01-JAN-2000 12:00:00 ET	**2451545.0**	**136.7535°**	**J2000**
01-JAN-2001 12:00:00 ET	2451911.0	136.7548°	
20-OCT-2001 12:00:00 ET	**2452203.0**	**136.7557°**	**MSP'01 Orbiter Arrival**
01-JAN-2002 12:00:00 ET	2452276.0	136.7559°	
22-JAN-2002 12:00:00 ET	**2452297.0**	**136.7559°**	**MSP'01 Lander Arrival**
01-JAN-2003 12:00:00 ET	2452641.0	136.7564°	
20-DEC-2003 12:00:00 ET	**2452994.0**	**136.7571°**	**MSP'03 Lander Arrival**
01-JAN-2004 12:00:00 ET	2453006.0	136.7571°	
01-JAN-2005 12:00:00 ET	2453372.0	136.7580°	
01-JAN-2006 12:00:00 ET	2453737.0	136.7590°	
17-JUL-2006 12:00:00 ET	**2453934.0**	**136.7596°**	**MSP'05 Lander Arrival**
01-JAN-2007 12:00:00 ET	2454102.0	136.7603°	
01-JAN-2008 12:00:00 ET	2454467.0	136.7614°	
01-JAN-2009 12:00:00 ET	2454833.0	136.7623°	

From this data it is apparent that the IAU-vector and the Equinox vector rotate at approximately 0.001 degrees per year with respect to one another.

**Figure 8:
Relationship Between Prime Meridian and Equinox
(shown at Epoch J2000)**



Right Ascension of Arbitrary point:

$\alpha_{EQU} = \alpha_{PM} + V$

Where:

$V = \Psi + W$

$W = 176.901 + 350.8919830d$ (deg)

Conclusions

The purpose of this memo is to update the Mars Observer Planetary Constants document (Ref 1) and properly define the relationships between the Mars Mean Equator and Equinox frame, the Mars Mean Equator and IAU-Vector frame, and the Mars Mean Equator and Prime Meridian frame. The parameter Ψ has been defined and calculated at various epochs over the coming decade for use by Mars mission planners.

It is important to note that the use of the Mars Equinox reference direction has been discouraged by the IAU as there is no standard for the mean orbit of Mars. Mission planners are encouraged to work with the IAU reference and to update software accordingly. If this is not possible or feasible, the data presented here will help to rotate between the various frames.

Suggested Acronyms

The following system is suggested as a convenient means of abbreviating the titles of the various coordinate systems. Because the names are long, they are often shortened or truncated which can lead to some ambiguity. The system suggested here is short enough to be useful, while maintaining all of the relevant information.

The model contains an abbreviation for each of the reference parameters of interest:

Center - Plane Direction Time

Where:

Central Bodies

E	Earth
S	Sun
M	Mars

Reference Planes

EME	Earth Mean Equator
ETE	Earth True Equator
EMO	Earth Mean Orbit (Ecliptic)
MME	Mars Mean Equator

Reference Directions

EQ	Equinox
IAU	IAU-vector
PM	Prime Meridian

Reference Times

2000	of Epoch J2000
E	of Epoch
D	of Date

Special Reference Frames

EME2000	Earth Mean Equator and Equinox of Epoch J2000
EMO2000	Earth Mean Orbit and Equinox of Epoch J2000

Suggested Acronyms

Inertial Coordinate Systems

E-EME2000	Earth-centered, Earth Mean Equator and Equinox of Epoch J2000
E-EMO2000	Earth-centered, Earth Mean Orbit and Equinox of Epoch J2000
S-EME2000	Sun-centered, Earth Mean Equator and Equinox of Epoch J2000
S-EMO2000	Sun-centered, Earth Mean Orbit and Equinox of Epoch J2000
M-EME2000	Mars-centered, Earth Mean Equator and Equinox of Epoch J2000
M-EMO2000	Mars-centered, Earth Mean Orbit and Equinox of Epoch J2000
M-MMEIAUE	Mars-centered, Mars Mean Equator and IAU-vector of Epoch
M-MMEPME	Mars-centered, Mars Mean Equator and Prime Meridian of Epoch
M-MMEEQE	Mars-centered, Mars Mean Equator and Equinox of Epoch

Non-Inertial Coordinate Systems

E-EMEEQD	Earth-centered, Earth Mean Equator and Equinox of Date
E-ETEEQD	Earth-centered, Earth True Equator and Equinox of Date
M-MMEIAUD	Mars-centered, Mars Mean Equator and IAU-vector of Date
M-MMEEQD	Mars-centered, Mars Mean Equator and Equinox of Date

Body Fixed (Non-Inertial) Coordinate Systems

E-EMEPMD	Earth-centered, Earth Mean Equator and Prime Meridian of Date
E-ETEPMD	Earth-centered, Earth True Equator and Prime Meridian of Date
M-MMEPMD	Mars-centered, Mars Mean Equator and Prime Meridian of Date

Appendix

MASL implementation of Mars Mean Orbit

To properly define the Mars Mean Equator and Equinox system, the Mars Mean Orbit plane must be defined. The calculation in the memo used the osculating orbit from DE405, which is not the mean orbit. Given below is one definition that can be used for comparison, and this is also the definition that is implemented in the JPL Mission Analysis Software Library (MASL).

Reference 4 provides the only generally accepted model for the mean orbit of Mars, which is a set of classical orbit elements referred to the Earth Mean Orbit and Equinox of Epoch B1950. These are:

$$
\begin{aligned}
a &= 227941000.0 \quad \text{(km)} \\
e &= 0.09335891275 + 0.000091987T - 0.000000077T^2 \\
i &= 1.85000 - 0.00821T - 0.00002T^2 \quad \text{(deg)} \\
\Omega &= 49.17193 - 0.29470T - 0.00065T^2 \quad \text{(deg)} \\
\omega &= 285.96668 + 0.73907T + 0.00047T^2 \quad \text{(deg)} \\
M &= 169.458720 + 0.5240207716d + 0.0001825972T^2 \\
&\quad + 0.0000011944T^3
\end{aligned}
$$

where:

d = ephemeris days from the reference epoch (JED 2433282.5, 01-Jan-1950)
T = d/36525, Julian centuries from the reference epoch.

These elements are described as polynomial functions of time due to the precession of the orbit with time. To obtain the Mars mean orbit at the J2000 epoch, utilize:

d = 2451545.0 - 2433282.5 = 18262.5 days
T = 0.5 Julian centuries

This will provide the orbit elements at the epoch J2000 in Earth Mean Orbit and Equinox of Epoch B1950 coordinates. To get these parameters into the EME2000 system, these classical orbit elements must first be converted to a cartesian state, then rotated to Earth Mean Equator and Equinox of Epoch B1950, and then rotated to EME2000 coordinates. Reference 5 provides a fixed rotation matrix for converting from Earth Mean Equator and Equinox of Epoch B1950 and Earth Mean Equator and Equinox of Epoch J2000:

$$
A = \begin{bmatrix} 0.9999256794956877 & -0.0111814832204662 & -0.0048590038153592 \\ 0.0111814832391717 & 0.9999374848933135 & -0.0000271625947142 \\ 0.0048590037723143 & -0.0000271702937440 & 0.9999881946023742 \end{bmatrix}
$$

This transformation matrix can be used to convert from any B1950 based ephemeris to a corresponding J2000 based ephemeris. It is important to note that this matrix is not absolutely precise for any given ephemeris, but rather represents an attempt to define a transformation that is sufficiently workable for an arbitrary ephemeris.

Now to convert any inertial vector from B1950 coordinates to the J2000 system:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{J2000} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{B1950}$$

The other necessary conversion is from Earth Mean Orbit and Equinox of Epoch B1950 to Earth Mean Equator and Equinox of Epoch B1950. The reference plane for the Ecliptic is taken to be the mean orbit of the Earth at the B1950 epoch, and thus the Z-axis is parallel to the angular momentum vector at that epoch. For both systems, the X-axis is defined to be the mean vernal equinox of the Earth at B1950, so that the coordinate systems are related by a single rotation about that axis. This angle of rotation is called the mean obliquity at the B1950 epoch and is given by Sturms (Ref.4) as:

$$\varepsilon_{B1950} = 23.4457889 \text{ deg}$$

Without going through the details, the mean orbit of Mars can be calculated at the J2000 epoch in EME2000 coordinates to be:

```
orbit pole  =  (0.024566, -0.416780, 0.908676)
```

This vector can also be obtained directly in QUICK using the PLOPOL function with the built-in analytic ephemeris described above. Recall that the Mars pole (at the J2000 epoch) has previously been defined:

```
mars pole  =  (0.446161, -0.406246,  0.797437)
```

(QUICK will calculate this via the PLPOLE function) and that from this, the IAU-vector (at the J2000 epoch) can be calculated:

```
iau vector  =  j2000 pole X mars pole

iau vector  =  (0.673258, 0.739408,  0.000000)
```

Now that the orbit pole of Mars is defined, the Vernal Equinox is defined as cross product between the Mars planet pole (of-date, previously defined at the epoch of J2000) and the orbit pole.

```
vernal equinox  =  mars pole X orbit pole

vernal equinox  =  (-0.086432, -0.906432 -0.413414)
```

So now the angle Ψ (at the epoch J2000) can be defined as the angle between the Vernal Equinox and the IAU-vector (at epoch J2000), and can be calculated via the dot product of the vectors.

$$\cos\Psi = \text{iau vector} \bullet \text{vernal equinox} = -0.728414$$

$$\Psi = 136.7536 \text{ deg}$$

This is a difference of 0.0001 deg with respect to the value quoted in the memo, where the mean orbit is taken to be the osculating orbit at the J2000 epoch.

Comments

The calculations in this appendix are documented in detail for several reasons. First, to show that the mean orbit of Mars is not well defined. The only reliable definition of the mean orbit of Mars is an aging set of polynomial coefficients in a coordinate system that cannot be precisely rotated to the J2000 standard. It is for this reason that the IAU recommends that an equinox based system not be used for planets other than the Earth.

Secondly, to allow these results to be easily reproduced in the future. Where possible, every equation and expression has been defined to allow ease of understanding, and traceability.

Third, to provide a transformation that exactly matches the MASL software that is used extensively for just this purpose at JPL. The numbers in this appendix can be easily and exactly reproduced by the MASL software set.

And finally, to show that the choice of osculating Mars orbit parameters to approximate the Mars Mean Orbit does not produce a significant discrepancy when compared to the Sturms definition. The value of Ψ at the J2000 epoch differs by only 0.0001 deg depending on which definition is utilized in the calculation. This is useful in that the osculating orbit parameters are readily available at any given epoch in a convenient coordinate system, based on the available ephemeris. The mean orbit definition provided by Sturms is difficult to rotate to a well-defined coordinate system.

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